ASTEROIDS, T. Gehrels, (ed.),
Univ. of Arizona Press (1979)

EARTH-CROSSING ASTEROIDS:

ORBITAL CLASSES, COLLISION RATES WITH EARTH, AND ORIGIN

E. M. SHOEMAKER, J. G. WILLIAMS, E. F. HELIN, AND R. F. WOLFE
California Institute of Technology

The term Earth-crossing asteroid is here taken to mean a minor planet on an orbit which, as a consequence of secular perturbations, can intersect the orbit of the earth. Known classes of Earth-crossing asteroids include Aten asteroids ($a < 1.0$ AU, $Q > 0.983$ AU), Apollo asteroids ($a \geqslant 1.0$ AU, $q \leqslant 1.017$ AU), and Amor asteroids ($a > 1.0$ AU, 1.017 AU $< q \leqslant 1.3$ AU). All three known Atens, all but one of the 23 known Apollos, and half of the 20 known Amors are Earth crossers. The total population of Earth-crossing asteroids to $V(1,0) = 18$ is estimated at $\sim 1.3 \times 10^3$, of which $\sim 8\%$ are Atens, $\sim 50\%$ are Apollos, and $\sim 40\%$ are Amors. A wide variety of physical types is represented among the Earth crossers, including four objects with UBV colors in the C field, several S-type objects, and several objects of distinctive colors that fall outside the C and S fields. The Earth crossers probably are of diverse origin; some probably have been derived from a residual population of old Mars crossers, some from widely separated regions of the main asteroid belt, and some from short-period comets. The principal sources appear to be extinct comet nuclei and collision fragments from regions in the main asteroid belt bordering the $\dot{\nu}_5$ and $\dot{\nu}_6$ secular resonances and the 3:1 and 5:2 commensurabilities with Jupiter. The present collision rate of Earth-crossing asteroids with the earth is estimated at ~ 3.5 objects, to absolute magnitude 18, per million years. Comparison of this rate with the record of impact cratering on the earth and the moon suggests that the present population of Earth crossers may be larger than the average population over the past 3.3 billion years.

The term Earth-crossing is here applied to include all asteroids on orbits which, as a consequence solely of secular perturbations, can intersect the orbit of the Earth. This definition is, at once, both broader and more exclusive than common past usage, where "Earth-crossing asteroid" has been taken to mean an object on an orbit that currently overlaps the orbit of Earth. Rigorous application of the concept of Earth- or planet-crossing orbits has been dependent, however, on the development of appropriate theoretical techniques (Williams 1969) or on practicable numerical integration by high-speed computer to solve the secular variation of asteroid orbits. Only recently have sufficient calculations been completed, primarily by Williams (1979 and unpublished), to confidently identify the known Earth-crossing asteroids.

It has long been recognized (Öpik 1951) that an asteroid orbit which overlaps the orbit of a planet may, as a result of secular advance of the apsides, intersect the orbit of the planet. The topological relationship between overlapping orbits has two possible states: linked and unlinked. If overlap of a planetary orbit of low eccentricity by a highly eccentric asteroid orbit is temporally continuous and deep, the two orbits must be linked (looped through one another like two links of a chain) when the argument of perihelion of the asteroid orbit ω is 0 and π. At $\omega = \pi/2$ and $3\pi/2$, on the other hand, the two orbits are unlinked. In a complete cycle of advance of ω, therefore, the transition between linked and unlinked states occurs four times; at each of these transitions the two orbits must intersect. Intersections of crossings, as given by solution of the polar equation of the ellipse, occur at

$$\omega = \cos^{-1} \pm \frac{1}{e}\left[\frac{a(1-e^2)}{\rho} - 1\right] \tag{1}$$

where a is the semimajor axis of asteroid orbit, e is the eccentricity of asteroid orbit at time of each intersection, and ρ is the radius to the planet's orbit along the line of nodes at the time of intersection. Because, in general, the planet's orbit is not circular and its eccentricity also varies as a consequence of secular perturbation, ρ will have four different values in one cycle of ω, which must be found by simultaneous solution of Eq. (1) and the polar equation for the elliptical orbit of the planet; e_0 and ω_0, the eccentricity and argument of perihelion of the planet's orbit, must be known at the time of each crossing. For a perfectly circular orbit of Earth ($e_0 = 0$) all values of ρ are 1.0 AU; at the other extreme of e_0, 0.933 AU $\leqslant \rho \leqslant$ 1.067 AU. In the case of continuous deep overlap of orbits, the four crossings given by Eq. (1) can fail to occur only if ω librates within a restricted range. If the overlap of the orbit of an asteroid with that of a planet is sufficiently shallow, on the other hand, less than four crossings may occur in a complete cycle of advance at ω. Crossings can be missed when ω and ω_0, the argument of perihelion of the planet, have the same phase. Secular variation of either e or e_0, moreover, can lead to loss of overlap during parts of the cycle of ω. At and very close to the times of any crossing there is a finite and calculable probability of collision of the asteroid with the planet unless encounter is prevented by exact average commensurability between the mean motion of the asteroid and mean motion of the planet.

Secular perturbations cause periodic oscillations in e and γ, the inclination of an asteroid orbit to the invariable plane, sometimes in a, and also in e_0 and γ_0, the inclination of the orbit of the planet, in addition to changes in ω. As a consequence of these perturbations, the orbits of less than half of the known Earth-crossing asteroids overlap the orbit of Earth continuously. Orbits of the other known Earth crossers overlap the orbit of the earth part of the time and part of the time lie entirely outside the orbit of Earth, primarily as a result of secular variation of e. Similarly, orbits which lie entirely inside Earth's orbit part of the time are possible, though no objects have yet been found on such orbits. Intersections of asteroid orbits with the orbit of Earth can occur which are associated with transitions from the condition of overlap to the condition of nonoverlap and also the reverse transitions. The intersections occur when r_a, the radius to the node of the asteroid orbit, equals ρ.

Secular variation of e, as a general rule, is correlated in phase with secular variation of ω. Most commonly, e is at a maximum at $\omega = \pi/2, 3\pi/2$. If the amplitude of oscillation of e is relatively small, periodic oscillation of r_a, for ascending and descending nodes, occurs mainly as a result of secular advance of ω. This type of oscillation of r_a is illustrated for the asteroid 2062 Aten in Fig. 1. If the asteroid is a relatively deep crosser, as is the case for Aten, four crossings of r_a at ρ will occur in each cycle of ω. This type of Earth-crossing asteroid is referred to here as a *quadruple crosser.* If the amplitude of oscillation of e is sufficiently high, however, the orbit of the asteroid can change from a condition of nonoverlap, at $\omega = 0,\pi$, to relatively deep overlap at $\omega = \pi/2, 3\pi/2$. In this case, two crossings can occur with each $\pi/2$ advance of ω, one controlled mainly by the large change of e and one mainly by the advance of ω. This type of oscillation of r_a is illustrated for the asteroid 1974 MA in Fig. 2; a total of eight crossings occur in each cycle of ω. We refer to this type of asteroid as an *octuple crosser.* 1580 Betulia was the first asteroid recognized to have this type of crossing behavior (Wetherill and Williams 1968).

In some cases, where γ is sufficiently high, secular perturbations do not lead to continuous advance in ω. Instead, ω librates around $\pi/2$ or $3\pi/2$; large periodic oscillation of e occurs during the libration cycle, with e reaching both a maximum and a minimum at the central value of ω. For deep crossers, the combined effect of oscillation of both e and ω produces four crossings of r_a at ρ in each libration cycle of ω. We refer to asteroids exhibiting this type of crossing behavior, such as 1973 NA (Fig. 3), as *quadruple crossing ω*

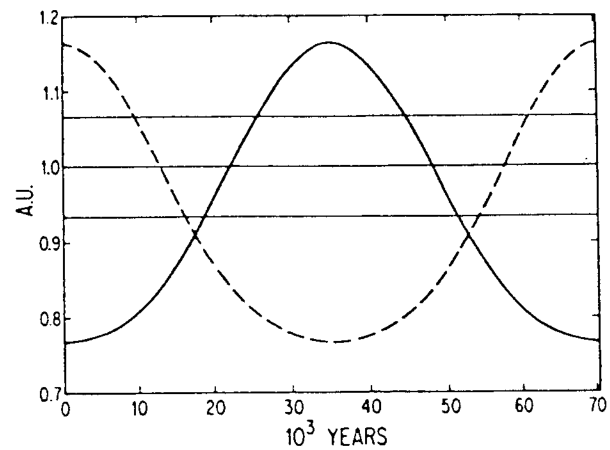

Fig. 1. Secular variation of radius to the node of the orbit of 2062 Aten for the zero-order state (forced oscillations in e are neglected). One cycle of advance of ω is represented, starting at $\omega = 0$. Ascending node shown with solid line and descending node with dashed line. Extreme values of aphelion and perihelion distances of the earth are shown by horizontal lines lying equal distances above and below 1 AU. Aten is an example of a quadruple crosser.

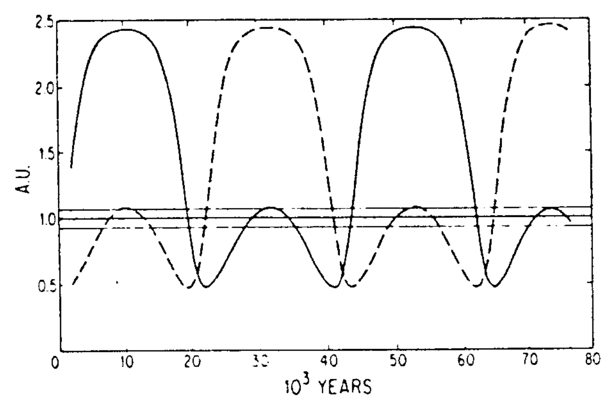

Fig. 2. Secular variation of radius to the node of the orbit of 1974 MA, based on forward integration of the motion of the asteroid. About 1.5 cycles of advance of ω are represented. Ascending node shown with solid line and descending node with dashed line. Extreme values of aphelion and perihelion distances of the earth are shown by horizontal lines lying equal distances above and below 1 AU. 1974 MA is an example of an octuple crosser.

librators.

A fourth type of crossing behavior is exhibited by Earth-crossing asteroids that librate about the 3:1 commensurability with Jupiter. The 3:1 resonant perturbation causes relatively high-frequency oscillation in both a

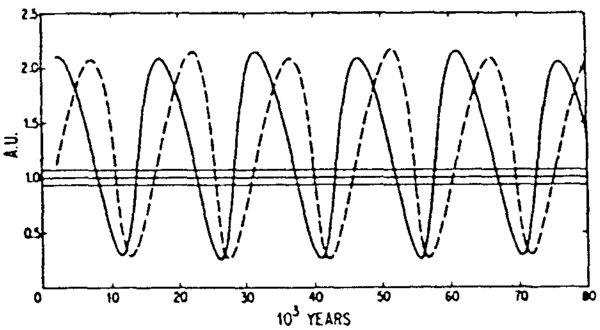

Fig. 3. Secular variation of radius to the node of the orbit of 1973 NA, based on forward integration of the motion of the asteroid. About five cycles of libration of ω are represented. Ascending node shown with solid line and descending node with dashed line. Extreme values of aphelion and perihelion distances of the earth are shown by horizontal lines lying equal distances above and below 1 AU. 1973 NA is an example of a quadruple crossing ω librator.

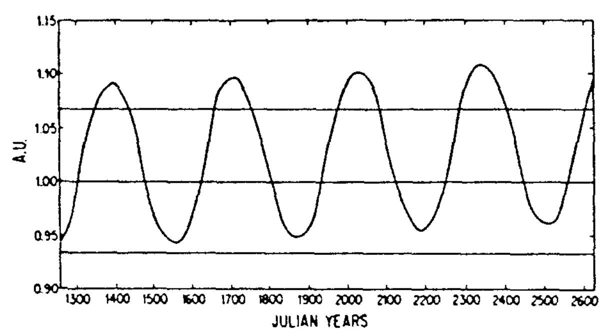

Fig. 4. Secular variation of radius to the ascending node of the orbit of 1915 Quetzalcoatl, based on forward and backward integrations of the motion of the asteroid by Marsden. About four cycles of libration of the mean motion of the asteroid about the 3:1 commensurability are represented. Extreme values of aphelion and perihelion distances of the earth are shown by horizontal lines lying equal distances above and below 1 AU. 1915 Quetzalcoatl is an example of a supercrosser.

and e, which can lead, in turn, to relatively high-frequency oscillation between conditions of overlap and nonoverlap of the orbit of the asteroid with the orbit of Earth. Slightly more than four cycles of oscillation of r_a in 1400 years and nine crossings of the orbit of Earth by the ascending node are illustrated for the asteroid 1915 Quetzalcoatl in Fig. 4. Asteroids exhibiting this type of crossing behavior are referred to here as *supercrossers.* Marsden (1970) has carried out 1400 year integrations of the motions of the supercrossers 1915 Quetzalcoatl and 887 Alinda, but much more numerical study must be done before the crossing behavior of these asteroids is fully understood.

I. CLASSES OF EARTH-CROSSING ASTEROIDS

The Earth-crossing asteroids are divided, for purposes of further discussion in this chapter, into three groups on the basis of their present osculating orbital elements: (1) Aten asteroids, (2) Apollo asteroids, and (3) Earth-crossing Amor asteroids. This classification has been used primarily because of its simplicity. However, the stable characteristics of the orbits of Apollo asteroids that overlap Earth's orbit part of the time and of the orbits of Earth-crossing Amors are not basically different. These two orbital classes are distinguished chiefly by the present phase of the cycle of variation of their perihelion distance. Most Earth-crossing Amors are only shallow Earth crossers, on the other hand, whereas the majority of Apollos are deep crossers. The orbits of most Earth-crossing Amors, moreover, overlap the orbit of the earth only a small fraction of the time. Therefore it seems useful to retain the traditional distinction between Apollo and Amor asteroids.

Aten Asteroids

Aten asteroids (Helin *et al.* 1978) have orbits with semimajor axes less than 1 AU which overlap the orbit of Earth near their aphelia. This class includes all asteroids with $a < 1.0$ AU and the aphelion distance of the asteroid, $Q \geqslant 0.983$ AU, where 0.983 AU is the present perihelion distance of Earth. Since Aten, the first member of this class, was discovered in 1976 (Helin and Shoemaker 1977) two more have been found (Table I). All three objects are relatively deep quadruple crossers whose orbital overlap with Earth's orbit is continuous or nearly continuous.

The total number of Aten asteroids to V(1,0) = 18, as will be shown, probably is on the order of 100. Many of these Atens may be expected to exhibit only part-time orbital overlap with Earth, just as many Apollos exhibit only part-time overlap at perihelion. Hence it may be expected that there are Earth crossers with orbits that are, at present, entirely inside the orbit of the Earth, just as there are Earth-crossing Amors with current osculating orbits entirely outside the orbit of the earth. As a rough guess, the population of this undiscovered class of Earth crossers with current aphelia less than 0.983 AU may be a few tens of objects to V(1,0) = 18. Wetherill (1979) has estimated the total population of asteroids with orbits inside Earth's at one to a few percent of the population of Apollos.

Apollo Asteroids

Apollo asteroids are defined by $a \geqslant 1.0$ AU, and perihelion distance, $q \leqslant$ 1.017 AU, where 1.017 AU is the present aphelion distance of the earth. Apollo, the first such asteroid discovered, was found by K. Reinmuth in 1932. The orbits of Apollos overlap the orbit of Earth in the region of perihelion. Out of 22 known Apollos with reasonably well-determined orbits, 21 are Earth-crossing (Table I). 1866 Sisyphus is a doubtful crosser; on a

TABLE I

Earth-crossing Asteroids: Crossing Characteristics and Collision Parameters

	Orbital overlap with Earth	Linkage with Earth's orbit F_ℓ[a]	Linkage state[b]	Earth-crossing class[c]	Orbital elements at $\rho = 1.0$ AU[d] a (AU)	e	γ (Deg)	$\lvert dr_a/dt \rvert$ $\rho = 1.0$ AU[e] (10^{-4} AU yr^{-1})	T_ℓ[f] (10^4 yr)	P_s[g] (10^{-9} yr^{-1})	P_o[h] (10^{-9} yr^{-1})
ATEN ASTEROIDS											
1976 UA	Continuous	0.55	L	Quad.	0.844	0.424	6.27	0.28	9.54	14	14
2100 Ra-Shalom	Continuous	0.54	L	Quad.	0.832	0.465	13.1	0.32	9.91	6.3	6.7
2062 Aten	Continuous	0.73	L	Quad.	0.966	0.237	17.9	0.20	7.04	6.9	6.4
APOLLO ASTEROIDS											
1978 SB	Continuous	0.89	L	Quad.	2.164	0.888	10.5	32	0.83	0.41	1.2
1566 Icarus	Continuous	0.70	L	Quad.	1.078	0.828	20.2	2.2	5.30	1.6	2.0
1974 MA	Part time	0.61	L	Oct.	1.757	{0.446 0.772	53.4 32.8	0.66 3.6}	4.23	0.93	2.5
2101 Adonis	Continuous	0.95	L	Quad.	1.873	0.732	1.96	7.3	1.58	2.9	6.3
1864 Daedalus	Continuous	0.92	L	Quad.	1.461	0.640	15.9	2.1	3.69	1.0	1.6
1865 Cerberus	Continuous	0.80	L	Quad.	1.080	0.513	14.9	0.68	6.81	2.5	3.1
Hermes	Continuous	0.99	L	Quad.	1.639	0.622	5.62	2.6	2.57	2.2	3.7
1981 Midas	Part time	0.29	U	Q.O.L.	1.776	{0.586 0.652	45.5 41.3	0.42 0.53}	2.62	3.8	0.7
1862 Apollo	Continuous	0.91	L	Quad.	1.470	0.505	6.14	1.3	3.00	3.5	4.8
2063 Bacchus	Continuous	0.94	L	Quad.	1.077	0.320	8.78	0.49	4.41	7.4	7.7
1959 LM	?	(0.8)	L	?	(1.34)[i]	(0.379)[i]	(3.3)	--	--	--	(13)[i]
1685 Toro	Continuous	(0.8)	L	Quad.?	1.368	0.438	9.15	0.78	(3)[j]	(4)[j]	(4.2)[j]
2135 1977 HA	Part time	0.76	U	Q.&O.	1.600	(0.490)	(23.3)	(0.43)	(3.97)	(2.0)[k]	(1.5)[k]
6743 P-L	Continuous	0.73	U	Quad.	1.620	0.476	6.81	0.82	2.87	4.1	4.8
1620 Geographos	Continuous	0.84	U	Quad.	1.245	0.351	14.2	0.39	5.21	3.8	3.9
1976 WA	Part time	0.74	L	Oct.	2.407	{0.589 0.700	32.3 17.0	0.8 5.1}	1.19	(1.8)[k]	(2.1)[k]
1950 DA	Continuous	0.83	L	Quad.	1.683	0.535	10.7	1.3	2.66	1.8	2.4

TABLE I

Earth-crossing Asteroids: Crossing Characteristics and Collision Parameters

	Orbital overlap with Earth	Linkage with Earth's orbit F^a_ℓ	Linkage state[b]	Earth-crossing class[c]	Orbital elements at $\rho = 1.0$ AU[d] a (AU)	e	γ (Deg)	$\lvert dr_a/dt \rvert$ $\rho = 1.0$ AU[e] (10^{-4} AU yr^{-1})	T_c[f] (10^4 yr)	P_s[g] ($10^{-9} yr^{-1}$)	P_ρ[h] ($10^{-9} yr^{-1}$)
ATEN ASTEROIDS											
1976 UA	Continuous	0.55	L	Quad.	0.844	0.424	6.27	0.28	9.54	14	14
2100 Ra-Shalom	Continuous	0.54	L	Quad.	0.832	0.465	13.1	0.32	9.91	6.3	6.7
2062 Aten	Continuous	0.73	L	Quad.	0.966	0.237	17.9	0.20	7.04	6.9	6.4
APOLLO ASTEROIDS											
1978 SB	Continuous	0.89	L	Quad.	2.164	0.888	10.5	32	0.83	0.41	1.2
1566 Icarus	Continuous	0.70	L	Quad.	1.078	0.828	20.2	2.2	5.30	1.6	2.0
1974 MA	Part time	0.61	L	Oct.	1.757	{0.446 0.772	53.4 32.8	0.66 3.6 }	4.23	0.93	2.5
2101 Adonis	Continuous	0.95	L	Quad.	1.873	0.732	1.96	7.3	1.58	2.9	6.3
1864 Daedalus	Continuous	0.92	L	Quad.	1.461	0.640	15.9	2.1	3.69	1.0	1.6
1865 Cerberus	Continuous	0.80	L	Quad.	1.080	0.513	14.9	0.68	6.81	2.5	3.1
Hermes	Continuous	0.99	L	Quad.	1.639	0.622	5.62	2.6	2.57	2.2	3.7
1981 Midas	Part time	0.29	U	Q.O.L.	1.776	{0.586 0.652	45.5 41.3	0.42 0.53 }	2.62	3.8	0.7
1862 Apollo	Continuous	0.91	L	Quad.	1.470	0.505	6.14	1.3	3.00	3.5	4.8
2063 Bacchus	Continuous	0.94	L	Quad.	1.077	0.320	8.78	0.49	4.41	7.4	7.7
1959 LM	?	(0.8)	L	?	(1.34)[i]	(0.379)[i]	(3.3)	--	--	--	(13)[i]
1685 Toro	Continuous	(0.8)	L	Quad.?	1.368	0.438	9.15	0.78	(3)[j]	(4)[j]	(4.2)[j]
2135 1977 HA	Part time	0.76	U	Q.&O.	1.600	(0.490)	(23.3)	(0.43)	(3.97)	(2.0)[k]	(1.5)[k]
6743 P-L	Continuous	0.73	U	Quad.	1.620	0.476	6.81	0.82	2.87	4.1	4.8
1620 Geographos	Continuous	0.84	U	Quad.	1.245	0.351	14.2	0.39	5.21	3.8	3.9
1976 WA	Part time	0.74	L	Oct.	2.407	{0.589 0.700	32.3 17.0	0.8 5.1 }	1.19	(1.8)[k]	(2.1)[k]
1950 DA	Continuous	0.83	L	Quad.	1.683	0.535	10.7	1.3	2.66	1.8	2.4

[d]Except as otherwise noted, all values of e and γ are for the zero order state (i.e. only free oscillations of e and γ are considered) derived by interpolation from computer runs based on a first order theory of secular perturbation (Williams 1969). Node and γ are referred to the invariable plane. Values e and γ bar ω librators, for 1685 Toro, and for 1580 are average values obtained from numerical integration of the motion of the asteroid by Williams; for Quetzalcoatl e and γ were derived by us from a numerical integration by Marsden (personal communication, 1979).

[e]$|dr_a/dt|$ at $\rho = 1.0$ AU is the derivative with respect to time of the radius to the node when the radius is equal to 1.0 AU. This derivative is obtained by interpolation from computer runs indicated in footnote d.

[f]For quadruple and octuple crossers, T_c is the mean period of a complete cycle of advance of ω; for ω librators, T_c is the mean period of libration of ω; for supercrossers, T_c is the period of oscillation of r. The values of T_c are derived from computer runs indicated in footnote d.

[g]P_s is probability of collision with Earth based on Eqs. (6) and (7).

[h]P_0 is probability of collision with Earth based on the equations of Öpik (1951), using the orbital elements listed in this table. For octuple crossers, the probabilities obtained from the two sets of e and γ were summed, and, for ω librators, the two solutions for the probability were averaged.

[i]Orbital elements based on short arc and not accurately known. Values listed for e and γ are the current osculating values and are used to obtain an approximate solution for P_0.

[j]Toro currently has a commensurable mean motion with the Earth. T_c is estimated from a numerical integration of the motion of Toro that spans only a fraction of the period of oscillation of r_a. P_s and P_0 are calculated by assuming that the present state of commensurable motion is short lived.

[k]P_s and P_0 listed for 1977 HA and 1976 WA should be considered lower limits, because forced oscillations of e will occasionally lead to very low values of $|dr_a/dt|$ at $\rho = 1.0$ AU.

[ℓ]Collision parameters and collision probabilities for Quetzalcoatl are based on numerical integration of motion spanning only a small fraction of the secular variation of ω, hence the true mean collision parameters are not known.

278,000 year integration of the motion of this object by Williams, r_a did not become smaller than 1.07 AU. One other Apollo whose orbit has been determined only from a short arc, 1959 LM, very probably is an Earth crosser. Among the 21 established Earth crossers, 13 exhibit continuous or nearly continuous orbital overlap with the Earth and eight have part-time overlap. All 13 with continuous orbital overlap are quadruple crossers or a probable quadruple crosser. Among the Apollos that exhibit part-time overlap with the earth, four are octuple crossers, one is a quadruple crosser part of the time and an octuple crosser part of the time, and three are quadruple crossing ω librators.

Amor Asteroids

Amor asteroids have been defined simply on the basis of perihelion distance, 1.017 AU $< q \leqslant$ 1.3 AU (Shoemaker and Helin 1978). These are objects that make relatively close approaches to the earth but do not, at present, overlap the earth's orbit. Traditionally, the name Amor, an asteroid discovered in 1932 by E. Delporte, has been given to this class, even though Eros, discovered in 1898, is a much better known member. The upper bound of q at 1.3 AU is arbitrary; it was chosen near a minimum in the radial frequency distribution of q for discovered objects. Somewhat less obviously, the distinction between Amor and Apollo asteroids is also rather arbitrary; some Apollos become Amors, and vice versa, as a consequence of secular perturbations. Had the Amor asteroid 1915 Quetzalcoatl been found in 1942 it would have been classed as an Apollo at the time of discovery. Of 20 known Amor asteroids, half are Earth-crossing, including 1221 Amor. Six Amors are quadruple crossers, three are supercrossers, and one is an octuple crosser (Table I). Only two of these objects, 1580 Betulia and 1915 Quetzalcoatl, are deep or even moderately deep Earth crossers, however.

II. LINKAGE STATES

For an asteroid orbit with deep overlap with Earth's orbit, constant e, and uniform $\dot{\omega}$, the fraction of time that the orbits are linked, F_{ℓ}, is given approximately by

$$F_{\ell} \approx \frac{2}{\pi} \cos^{-1} \left| \frac{1}{e} \left[\frac{a(1-e^2)}{\bar{\rho}} - 1 \right] \right| \qquad (2)$$

where $\bar{\rho} = 1$. The linkage fraction, F_{ℓ}, is close to 1 when the semilatus rectum, $a(1-e^2)$, is close to $\bar{\rho}$; F_{ℓ} will be much less than 1 when the semilatus rectum is either much greater or much less than $\bar{\rho}$.

If Apollos and Atens, the asteroids with orbits which currently overlap Earth's orbit, are combined, the mean predicted linkage fraction, taking into account secular variation of e and nonuniform $\dot{\omega}$, is 0.74. The observed degree of linkage is 16/23 or 0.70 (Table I), in satisfactory agreement with prediction. This close agreement suggests that there is no strong selection effect in the discovery of Earth-crossing asteroids with regard to the present radii to the nodes of the asteroids.

III. COLLISION RATES WITH EARTH

All known Earth-crossing asteroids, as defined in this chapter, have a finite probability of collision with Earth. Only in the case of 1685 Toro has a commensurable mean motion been found that might preclude collision at times of orbit intersection (Danielsson and Ip 1972; Janiczek *et al.* 1972; Ip and Mehra 1973). As shown by Williams and Wetherill (1973), however, even if Toro is able to avoid Earth at the present time, close encounters with Mars will tend to displace Toro from deep resonance in about 3×10^6 yr. Its probability of collision with Earth can be decreased only slightly by its present commensurable motion.

Close encounters with the planets cause large changes in the orbits of the planet-crossing asteroids. Many such changes usually occur in the orbit of an Earth crosser before it meets its ultimate fate – collision or ejection from the solar system (Arnold 1964, 1965; Wetherill and Williams 1968). Because of these changes, the probability of collision with Earth at any one time in the orbital evolution of a given asteroid does not ordinarily represent its long-term average probability of collision. The sum of the present probabilities of collision with Earth of all Earth crossers, on the other hand, does approximate the present collision rate with Earth, as the total number of Earth crossers is large enough for this sum to be statistically stable.

We will assume here that the orbits of the discovered Aten, Apollo and Earth-crossing Amor asteroids represent a relatively unbiased sample of the orbits of the entire population in each class of Earth crossers. The mean present probabilities of collision for each class, estimated from the available sample of orbits, multiplied by the estimated population will provide an estimate of the present collision rate for objects in that class. An estimate of the present collision rate on Earth, then, will be obtained from the sum of the collision rates for all classes of Earth-crossing objects.

The collision rate on Earth may be expected to change only slowly with time. Although the orbits of individual Earth crossers are scrambled by close planetary encounters, the statistical distribution of orbital characteristics of the entire population of Earth crossers probably is nearly in steady state. The dynamical relaxation time of the Earth-crossing asteroid population may be estimated by the harmonic mean lifetime of Earth crossers against collision or ejection. As found from Monte Carlo studies of close encounter and collision, typical lifetimes for Earth crossers are on the order of a few tens of millions of years (Wetherill and Williams 1968; Wetherill 1976). The present collision rate on Earth may be representative of the rate for the past 10^7 yr and possibly much longer, if the population of Earth crossers is also approximately in equilibrium. We will test this possibility by comparison of the present collision rate with the geological record of impact on the earth.

An approximate theory for calculating the probability of collision of planet-crossing bodies was first developed by Öpik (1951). By breaking down the dynamics of encounter into a series of two-body approximations, Öpik greatly simplified the calculation of impact probability. One of the colliding bodies generally is taken to be of major planetary dimensions and one much smaller, and the motion of the two bodies near encounter is taken as linear, which leads to still further simplification. The conditions under which the errors introduced by these approximations remain small has been discussed in detail by Öpik (1976). In the development of Öpik's theory, the orbit of the major planet was initially taken as circular and a correction applied to the final equations to account for a nonzero mean e_0. Moreover, e and γ for the minor body were assumed constant and ω, over a sufficiently long period of time, was assumed uniformly distributed. This last assumption is equivalent to taking $\dot{\omega}$ as constant or independent of ω. In order to deal with collisions between asteroids, Wetherill (1967) refined Öpik's theory by introducing the finite eccentricity of both bodies at the outset of the development of the problem. The result is a more precise but significantly more complex set of equations.

As will be seen, Öpik's assumption of constant e and uniform distribution of ω commonly leads to significant error, especially in the case of ω librators. Now that the necessary calculations have been carried out for the secular variation of the orbital elements of the Earth crossers, it is appropriate to introduce further revisions in Öpik's theory to take account of these variations. Our development of the theory closely parallels that of Wetherill's (1967). In place of an explicit formula for $d\Delta/d\omega$ at the time of crossing introduced by Öpik (1951), where $\Delta = r_a - r_p$ and r_p is the radius to the nearest node of the orbit of the planet, and the assumption of Öpik of constant $d\omega/dt$, however, we let $d\Delta/dt$ remain a variable to be determined from secular perturbation theory. The resulting expression for probability of collision of an asteroid with a planet per unit time P_s is

$$P_s = \frac{1}{4\pi T_c} \Sigma \left| \frac{d\Delta}{dt} \right|_i^{-1} \frac{\tau_i^2 U_i \rho_i}{a_0^2 (1-e_{0i}^2)^{\frac{1}{2}} a_i^2 (1-e_i^2)^{\frac{1}{2}} \sin i} \quad (3)$$

where T_c is the period of oscillation of Δ, U is the encounter velocity of the asteroid at the sphere of influence of the planet, τ is the capture radius of the planet, i is the inclination of the asteroid orbit referred to the plane of the planet's orbit, and all elements subscripted i as well as $|d\Delta/dt|$ are the values at the times of crossing

$$U_i^2 = \frac{GM}{\rho_i^2} \left\{ a_i \frac{(2A_i - 1)}{A_i^2} + a_o \frac{(2A_{oi} - 1)}{A_{oi}^2} \right. \tag{4}$$
$$\left. -2 \left[a_i a_o (1 - e_i^2)(1 - e_{oi}^2) \right]^{\frac{1}{2}} (\cos i_i + \cot \alpha_i \cot \alpha_{oi}) \right\}$$

where G is the gravitational constant, M is the mass of the sun, A_i is the a_i/ρ_i, A_{oi} is the a_o/ρ_i, and

$$\cot^2 \alpha_i = \frac{A_i^2 e_i^2 - (A_i - 1)^2}{A_i^2 (1 - e_i^2)}, \quad \cot^2 \alpha_{oi} = \frac{A_{oi}^2 e_{oi}^2 - (A_{oi} - 1)^2}{A_{oi}^2 (1 - e_{oi}^2)} \tag{5}$$
$$\tau_i^2 = R^2 \left(1 + \frac{8 \pi^2 m}{R U_i^2 M}\right)$$

where R and m are respectively the radius and mass of the planet. For quadruple crossers and octuple crossers, T_c is the period of one cycle of advanced of ω; for quadruple crossing ω librators, T_c is the libration period; for supercrossers, T_c is provisionally taken as the libration period of a. The summation in Eq. (3) is to be carried out for the number of crossings in one period T_c, nominally four in the case of quadruple crossers, eight in the case of octuple crossers, and two in the case of supercrossers. Under conditions of shallow overlap of orbits, the number of crossings can be less than nominal. A rigorous evaluation of Eq. (3) would consist of a time-averaged value of P_s for all simultaneous combinations of a_i, e_i, i_i, e_{oi}, ρ_i and $|d\Delta/dt|_i$ that can occur at the times of crossing.

We are not yet prepared to offer a precise solution to Eq. (3) for any Earth crosser, but an approximate solution for deep crossers can be obtained by adopting $\rho = 1$ AU, which is very close to the average ρ for deep crossers. Mean values of a, e, i, and $|dr_a/dt|$ at $\rho = 1$ AU can then be estimated fairly readily from secular perturbation theory. Mean $|dr_a/dt|$ at $\rho = 1$ AU, moreover, is close to mean $|d\Delta/dt|$ at $\rho = 1$ AU. Octuple crossers and quadruple crossing ω librators exhibit two distinctly different types of crossing, and two sets of mean e, i, and $|dr_a/dt|$ have been estimated, one set for each type of crossing. Equations (3) and (4) then reduce to

$$P_s = \frac{\rho^2}{\pi T_c a_o^2 (1 - e_o^2)^{\frac{1}{2}}} \left[\left| \frac{dr_a}{dt} \right|_1^{-1} \frac{\tau_1^2 U_1}{a_1^2 (1 - e_1^2)^{\frac{1}{2}} \sin i_i'} \right. \tag{6}$$
$$\left. + \left| \frac{dr_a}{dt} \right|_2^{-1} \frac{\tau_2^2 U_2}{a_2^2 (1 - e_2^2)^{\frac{1}{2}} \sin i_2'} \right]$$

where e_0^2 is the mean squared eccentricity of Earth's orbit, $\sin i'$ is $(\sin^2\lambda + \sin^2\lambda_0)^{1/2}$, and $\sin^2\lambda_0$ is the mean squared sine of inclination of Earth's orbit to invariable plane, and subscripts 1 and 2 refer to the two distinct types of crossing;

$$U^2 = \frac{GM}{\rho}\left\{4 - \frac{1}{A} - \frac{1}{A_0} - 2\left[A A_0 (1-e^2)(1-e_0^2)\right]^{\frac{1}{2}}\cos i'\right\} . \quad (7)$$

The solution for P_s given by Eq. (6) must be divided by 2 to obtain the correct probability for collision of quadruple crossing ω librators and supercrossers.

First estimates of the collision parameters for each asteroid, which are T_c and mean a, e, γ, and $|dr_a/dt|$ at $\rho = 1.0$ AU, can be obtained for most quadruple and octuple crossers from secular perturbation theory by considering only the free oscillations of the orbital elements. These estimates are shown in Table I. The free oscillations, which remain when the eccentricities and inclinations of the perturbing planets are reduced to zero, are referred to here as the zero-order state. Forced oscillations, due to the finite and varying eccentricities and inclinations of the planets, produce dispersion of e, γ, and r_a, about the values found for the zero-order state. Crossings of very shallow crossers occur only as a consequence of the forced oscillations of e; in these cases the zero-order state does not yield values for the collision parameters. The collision parameters found for deep crossers from the zero-order state are close to the time-averaged values. But, for shallow crossers, the values obtained from the zero-order state will be displaced significantly from the time-averaged means. In particular, $|d\Delta/dt|$ at the time of crossing occasionally will approach zero, in the case of shallow crossers, as a result of $\dot{r}_a$ and $\dot{r}_p$ being in phase or as a consequence of the forced oscillations of e or of secular variation of e_0. Under this circumstance, Eqs. (6) and (7) no longer provide a satisfactory solution for P_s. Hence no solutions of P_s are shown for shallow crossers in Table I.

Cases where $|d\Delta/dt|$ becomes very small occur, in general, for asteroid orbits with only part-time overlap of Earth's orbit, and the fraction of crossings with small $|d\Delta/dt|$ tends to be roughly inversely proportional to the fraction of time that there is overlap. Thus, even though collision probabilities of shallow crossers tend to be high at times of crossing, the frequency of such crossings tends to be low, and the net collision probability generally is of the same order of magnitude as that of deep crossers. A detailed treatment of this problem will be given in a separate paper in preparation.

Collision parameters for ω librators, the supercrosser Quetzalcoatl, and also for the octuple crosser Betulia, which is in a resonant orbit, have been estimated from numerical integrations (Table I). Although based on a limited number of cycles of r_a, these estimates are believed to be nearly as close to the time averaged means as the collision parameters estimated from the

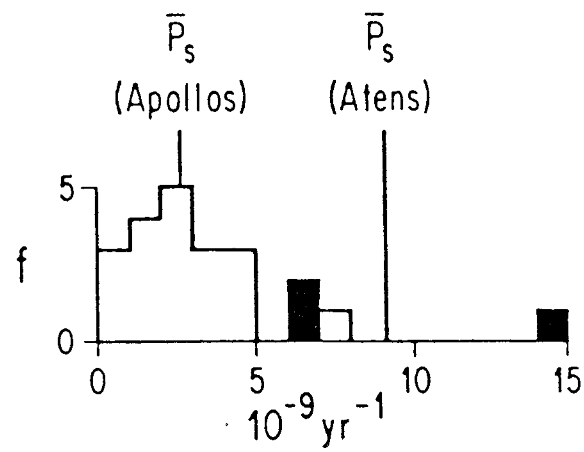

Fig. 5. Frequency distributions of collision probability with Earth, P_s, for Apollo (open bars) and Aten (solid bars) asteroids. Mean impact probabilities, $\bar{P}_s$, for Apollos and Atens are shown with vertical lines.

zero-order state.

It should be noted that the estimates of mean $|dr_a/dt|$ at $\rho = 1.0$ AU obtained either from the theoretical free oscillations of the orbital elements or from numerical integration tend to be maximum estimates. Low values of $|dr_a/dt|$ that occur at times of relatively shallow crossing (under conditions where the two-body linear approximation of Öpik is still good) are not represented in these estimated means. Thus all values of P_s calculated from Eqs. (6) and (7) (Table I) should be regarded as minimum estimates of the true collision probabilities.

Also shown in Table I are collision probabilities based on the equations of Öpik (1951), using the estimated mean orbital elements at $\rho = 1.0$ AU. It may be seen that P_0, the probability of collision with Earth based on Öpik's equations, is in good agreement with P_s for Aten asteroids. In some cases of Apollos and Earth-crossing Amors, however, p_0 differs from P_s by factors 2 to 5. Most of the large differences are found among the octuple crossers and ω librators, but a difference factor of more than 2.5 was found for the quadruple crossers Adonis and 1978 SB.

The frequency distributions of computed collision probabilities, P_s, for Apollos and Atens are shown in Fig. 5. Amors are not illustrated in this figure because solutions for Eqs. (6) and (7) can be obtained for only two Amors. The mean probability for collision with Earth is 9.1×10^{-9} yr^{-1} for Atens and 2.6×10^{-9} yr^{-1} for Apollos (where $P_s = 0$ has been adopted for the Apollo asteroid 1866 Sisyphus). Only a rough estimate can by made for mean P_s for the Amors. The orbits of some Amors overlap the orbit of Earth a very small fraction of the time, but they tend to have fairly high probability of

TABLE II

Collision rate with Earth of known classes of Earth-crossing asteroids

	Population to V(1,0)=18	Mean Collision Probability 10^{-9} yr^{-1}	Collision Rate to V(1,0)=18 10^{-6} yr^{-1}
Atens	~ 100	9.1	~ 0.9
Apollos	700 ± 300	2.6	1.8 ± 0.8
Earth-crossing Amors	~ 500	~ 1	~ 0.5
Total	~ 1300		~ 3.5

collision during these periods of overlap. The mean probability of collision with Earth for the known Earth-crossing Amors is not less than 0.4×10^{-9} yr^{-1} and probably is close to 1×10^{-9} yr^{-1}.

The population of Atens and Apollos combined is estimated by Helin and Shoemaker (1979) as 800 ± 300 to absolute visual magnitude V(1,0) = 18. As Atens constitute 3/26 or 12% of the combined set of known Atens and Apollos, the population of Atens is roughly estimated at ~ 100 to V(1,0) = 18, and the population of Apollos, by subtraction, is 700 ± 300. The population of Earth-crossing Amors can be estimated very roughly from the ratio of discovered Amors to discovered Apollos. A bias exists against discovery of Amors with large q and small a, however, both in systematic surveys and by accidental discovery (Helin and Shoemaker 1979). Therefore the ratio of discovered Amors to discovered Apollos, about 1:1, probably is lower than the ratio of the populations of these objects. Monte Carlo studies of evolution of orbits of Amor and Apollo asteroids suggest that the true ratio of the Amor population to the Apollo population is in the range of 1.5 to 3 (Wetherill 1979). Hence the Amor population to V(1,0) = 18 may be in the range of 1000 to 2000. Although half of the known Amors are Earth crossers, the Earth-crossing Amors tend to have relatively small q and the bias against their discovery is less than for the other Amors. We tentatively estimate the population of Earth-crossing Amors to absolute magnitude 18 at about 500.

When the estimated mean collision probabilities for the three known classes of Earth-crossing asteroids are multiplied by the estimates of the populations for each class (Table II) the total collision rate with Earth is found to be ~3.5 asteroids to absolute magnitude 18 per million years. On the average, one of these asteroids will be an Aten, two will be Apollos, and one half will be an Amor. To this list of colliding objects can be added the nuclei of active comets and the undiscovered class of Earth-crossing asteroids

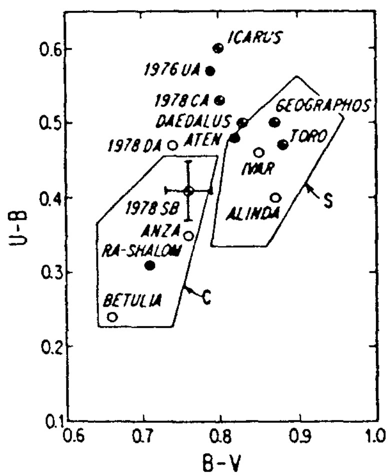

Fig. 6. UBV colors of Earth-crossing asteroids. Outer limits of color fields for *C*- and *S*-type asteroids are from Bowell *et al.* (1978). Error bars shown for 1978 SB (Bowell 1978) are probably fairly representative of errors of UBV observations of most other Earth-crossing asteroids. UBV data are from TRIAD data file (Part VII of this book). Solid dots are Atens, target symbols are Apollos, and open circles are Amors.

with orbits currently inside that of Earth. From a consideration of the observed flux of comets in the vicinity of Earth and unpublished measurements by E. Roemer of the magnitudes of comet nuclei when they are at large solar distances and relatively inactive, we estimate that the collision rate of still active comet nuclei is not more than about 10% of the collision rate of Earth-crossing asteroids. In our estimate of an upper bound for the collision rate of comets, a very large allowance is made for incompleteness of discovery of Earth-crossing comets. The contribution to the total collision rate of the undiscovered class of asteroids with very small orbits probably does not exceed ~ 5%.

IV. PHYSICAL CHARACTERISTICS OF EARTH-CROSSING ASTEROIDS

High-quality observations on physical characteristics have been obtained for about 40% of the known Earth-crossing asteroids. In six cases these observations have been made during the discovery apparition, all since January 1976. For some objects, only UBV observations have been made, but in addition, polarimetry, infrared radiometry, narrowband spectrophotometry and observations by radar have been obtained for a number of the Earth crossers. Diversity of UBV colors suggests that a variety of mineralogical compositions are represented among the Earth-crossing asteroids (Fig. 6). Four Earth crossers are *S*-type asteroids; two of these are Apollos and two are Amors. Four Earth crossers, including one Aten, one Apollo, and two Amors, have UBV colors in or on the boundary of the *C* field, as defined by Bowell *et al.* (1978). The colors of other Earth crossers lie outside the limits of the *C* and *S* fields, and some Earth crossers have UBV colors unlike those of any other asteroids. Two of these, the Aten asteroid 1976 UA and the Apollo asteroid 1566 Icarus, have closely similar UBV colors that are characterized by extreme values of U–B; Shoemaker and Helin (1978) suggested that the color and possibly other properties of the surfaces of these two asteroids may have been affected by close approaches to the sun.

Observations of Earth-crossing asteroids discovered in 1978 suggest that the concept of Earth crossers as predominantly *S*-type or high-albedo asteroids, based on earlier observations, must now be re-examined. When corrections are made for observational selection effects due to differences in albedo, it appears that the majority of Earth crossers, to any given size limit, may turn out to be dark objects.

Albedos have been estimated both from polarimetry and from infrared radiometry for six Earth crossers (Lebofsky *et al.* 1979). Three of these objects, Alinda, Aten and 1978 DA, have high visual geometric albedos (0.16 to 0.20), as derived from polarimetry as well as from radiometry, using a standard model of thermal inertia (Jones and Morrison 1974; Morrison 1977) for the surfaces of the asteroids. The other three objects, Betulia, Ra-Shalom and 1978 CA, have low polarimetric albedos; 1978 CA has a low radiometric albedo, as determined from the standard thermal inertia model, but radiometric albedos consistent with polarimetric observations are found for Betulia and Ra-Shalom only if the surfaces of these asteroids are assumed to be rocky (Lebofsky *et al.* 1978, 1979).

Perhaps the most remarkable solar system object, found in 1978, was 1978 SB. This Apollo asteroid, discovered by L. I. Chernykh, has an orbit somewhat like that of comet Encke and is probably the largest known Earth crosser. The UBV color of 1978 SB, measured by Bowell (1978), falls within the *C* color domain (Fig. 6); from measurements of 1978 SB reported by Bowell we estimated V(1,0) = 14.01, assuming a phase coefficient of 0.035 mag/deg. If a visual geometric albedo p_v near the mean for *C*-type asteroids is adopted for 1978 SB, its calculated diameter is greater than 10 km (Table III). As shown in Table III, if objects with UBV colors in the *C* field are assumed to have low albedo, following Bowell *et al.* (1978), then the bulk of the volume of photometrically observed Earth crossers is contained in these dark asteroids.

Clearly it is premature to draw firm conclusions about the relative abundance of difference physical types among the Earth-crossing asteroids from the small sample of physical observations now available. Many more observations are needed, especially polarimetric and radiometric observations of 1978 SB and 2061 Anza, the two largest Earth crossers. Discoveries and observations made in the past year do suggest, however, that the proportion of high-albedo asteroids among the Earth crossers is not as great as initially surmised. As will be seen, this leads to significant revision in the estimated present rate of impact crater production on the earth.

TABLE III

Estimated albedos and diameters of Earth-crossing asteroids

	p_V	Diameter (km)	Volume (km^3)	Reference
Asteroids with UBV colors in the *C* field				
1580 Betulia	0.04[a]	6.4	140	Lebofsky *et al.* (1978)
2061 Anza	(0.037)[b]	(9.5)	(450)	Bowell *et al.* (1978)
2100 Ra-Shalom	0.04[a]	3.4	21	Lebofsky *et al.* (1979)
1978 SB	(0.04)[a]	(10.4)	(590)	this chapter
			1200	
S-type asteroids				
887 Alinda	0.16_5^a	4.1	37	Zellner *et al.* (1974)
1620 Geographos	0.21[c]	2.0	4.2	Zellner *et al.* (1974)
1627 Ivar	(0.14)[b]	(7.0)	(180)	Bowell *et al.* (1978)
1685 Toro	$0.14_7{}^c$	6.0	110	Zellner *et al.* (1974)
			330	
Asteroids with UBV colors outside the *C* and *S* fields				
1566 Icarus	0.17_9^c	1.4	1.4	Zellner *et al.* (1974)
1865 Daedalus	(0.14)[b]	3.3	19	Bowell *et al.* (1978)
2062 Aten	0.20[a]	0.9	0.4	Morrison *et al.* (1976)
1976 UA	(0.14)[b]	(0.2)	(0.004)	Bowell *et al.* (1979)
1978 CA	0.06[d]	1.9	3.6	Lebofsky *et al.* (1979)
1978 DA	0.16[d]	0.9	0.4	Lebofsky *et al.* (1979)
			25	

[a] p_V based on polarimetry and radiometry.
[b] p_V assigned on the basis of UBV colors
[c] p_V based on polarimetry
[d] p_V based on radiometry

V. SOURCES OF EARTH-CROSSING ASTEROIDS

The origin of Earth-crossing asteroids has been a subject of long-standing debate. As first noted by Öpik (1951), the Apollo asteroids are not likely to have survived as Earth-crossing objects from the time of formation of the planets; their average lifetime against collision with the planets or ejection from the solar system is very much less than the age of the system. The initial population would necessarily have to be enormous and there would have been a steady, roughly exponential decline in the population throughout geologic time. The impact record of the earth and moon shows that such a decline did not occur in the last $\sim 3 \times 10^9$ yr. This same argument also applies to the Aten asteroids, which have still shorter lifetimes than Apollos. A few Earth-crossing Amors, with dynamic lifetimes on the order of 10^9 yr or longer could conceivably be surviving Earth planetesimals that have remained in the earth's neighborhood for 4.5 aeons. The vast majority of all Earth crossers, however, must have been injected from other regions of space into Earth-crossing orbits at relatively late times in solar system history.

Earth-crossing asteroids almost surely are of diverse origin; some probably have been derived from a residual population of old Mars crossers, some from widely separated regions of the main asteroid belt, and some from short-period comets. Öpik (1963) and Wetherill and Williams (1968) found that the population of Mars crossers would have to be several hundred times greater than the population of Apollos in order to maintain the Apollo population in quasi-steady state by deflection of Mars crossers. The actual ratio of Mars crossers to Apollos plus Atens, however, appears to be in the range of 10 to 60 (Shoemaker and Helin 1977; Helin and Shoemaker 1979). Part of the present population of Mars crossers, moreover, must consist of objects that have been injected into Mars-crossing orbits late in geologic time from the same source regions that yield the majority of Earth-crossing asteroids. Thus only a few percent of Apollos and Atens and a somewhat larger fraction of Earth-crossing Amors can be derived from the steadily dwindling supply of old Mars crossers (surviving Mars planetesimals).

Regions of the main asteroid belt bordering low-order commensurabilities and secular resonances are likely source regions for Earth-crossing asteroids. Large amplitude oscillations of certain orbital elements occur when asteroids are placed deep in these resonances; most such asteroids become Mars crossers and some would be occasional Earth crossers. The Kirkwood gaps at 2:1, 5:2, and 3:1 commensurabilities with Jupiter and zones centered on $\dot{\nu}_5$, $\dot{\nu}_6$ and $\dot{\nu}_{16}$ secular resonances of Williams (1969) are all greatly depleted in asteroids, a circumstance which suggests that asteroids have, indeed, been removed as a consequence of planetary encounters (Williams 1971). Zimmerman and Wetherill (1973), Williams (1973 *a, b*) and Scholl and Froeschlé (1977) have shown how meteorites could be delivered to Earth as collision fragments injected into the resonances from asteroids on stable orbits bordering either the 2:1 or 5:2 Kirkwood gaps or the $\dot{\nu}_5$ or $\dot{\nu}_6$ secular resonances. More recently Wetherill (1977, 1979) and Wetherill and Williams (1979) have examined the interplay between oscillation of orbital elements caused by the $\dot{\nu}_6$ resonance and the changes in orbit caused by close encounter with Mars. They found that collision fragments from asteroids bordering the $\dot{\nu}_6$ resonance can be delivered fairly efficiently into Earth-crossing orbits as a result of this interplay. Many objects initially injected into a shallow part of the resonance are later placed deep in the resonance as a result of Mars encounters (see the chapter by Wasson and Wetherill).

Once an asteroid becomes Earth-crossing, further orbital changes due to encounters with Earth and Venus tend to obscure its origin. It is of interest, nonetheless, to examine the present relationship of the known Earth crossers to resonances, in order to see whether these relationships provide any clues as to origin. Three Earth-crossing Amors currently librate around the 3:1 commensurability with Jupiter, but no known Earth crossers are near the 2:1 or 5:2 commensurabilities. This suggests that the 3:1 commensurability may play a substantial role in transferring asteroids into Earth-crossing orbits, although the precise mechanisms have not been studied. Probably there is a "synergistic" interaction between the resonant perturbations of the 3:1 commensurability and changes in orbit due to Mars encounter, as found by Wetherill (1977, 1979) for the $\dot{\nu}_6$ secular resonance. Regions of the main asteroid belt bordering both the 3:1 and 5:2 commensurabilities may be significant sources of Earth crossers.

With regard to the secular resonances, five Earth crossers are deep in the $\dot{\nu}_5$ resonance, one is in the $\dot{\nu}_{16}$ resonance, but none are known in the $\dot{\nu}_6$ resonance. Unlike the $\dot{\nu}_6$ resonance, $\dot{\nu}_5$ extends far into the region of the terrestrial planets. The occurrence of an Earth crosser in $\dot{\nu}_5$ does not, therefore, necessarily indicate its place of origin, especially for objects of small semimajor axis such as Icarus, Ra-Shalom and 1978 CA. These asteroids probably were placed in the $\dot{\nu}_5$ resonance by close planetary encounters at a late stage in their orbital evolution. On the other hand, Betulia and 1974 MA, which are also in $\dot{\nu}_5$, have semimajor axes of 2.20 and 1.76 AU, within the inner part of the main asteroid belt. Hence it is possible that they were derived from regions in the main belt close to $\dot{\nu}_5$. Daedalus is located in the $\dot{\nu}_{16}$ resonance, 1977 HA is close to $\dot{\nu}_{16}$ and 1974 MA is not only in $\dot{\nu}_5$ but also close to $\dot{\nu}_{16}$. It is not yet clear how $\dot{\nu}_{16}$ might have assisted the transfer of these objects into Earth-crossing orbits, but the asteroid belt is known to be depleted in the vicinity of $\dot{\nu}_{16}$. Probably the $\dot{\nu}_5$, $\dot{\nu}_6$ and $\dot{\nu}_{16}$ secular resonances all play significant roles in the transfer of collision fragments from main-belt asteroids into Earth-crossing orbits. On the basis of the present relationship of known Earth crossers to the resonances, $\dot{\nu}_5$ appears to be at least as important as $\dot{\nu}_6$ in the orbital evolution of Earth crossers, even though the density of potential parent belt asteroids is greater along the margin of $\dot{\nu}_6$.

From a quantitative assessment of the production of collision fragments, as well as the efficiency of their transfer to Earth-crossing orbits, Wetherill (1979) concluded that the best estimate of the yield of objects from the margin of the ν_6 resonance is an order of magnitude less than that required to maintain the Earth-crossing asteroid population. If all plausible sources in the main belt are considered, perhaps several tens of percent of the Earth crossers can be accounted for as collision fragments of belt asteroids. Most of the remaining Earth-crossing asteroids probably are derived from extinct short-period comets.

Most short-period comets have extremely short dynamical lifetimes, because they are Jupiter-crossing. Therefore they are unlikely to be captured by encounters with the terrestrial planets into orbits like those of the Earth-crossing asteroids. A few comets have aphelia inside the orbit of Jupiter, however, and one, P/Encke, has an aphelion as small as 4.1 AU, so that it does not pass within the sphere of influence of Jupiter. Evidently P/Encke has arrived in this orbit safe from Jupiter encounter as a consequence of nongravitational forces arising from the evaporation of volatile constituents during perihelion passages (Sekanina 1971). Historical decay of the nongravitational acceleration of P/Encke suggests that it might become extinct in a period as short as 60–70 yr (Sekanina 1972) leaving a kilometer-sized inactive nucleus that will be observed in the future as an Earth-crossing asteroid. A few other comets, in less stable orbits, also appear to be nearly extinct, including P/Arend-Rigaux and P/Neujmin 1 (see Kresák's chapter). Although the Jupiter-crossing object Hidalgo has been asteroidal in appearance at all times that it has been observed, its unusual orbit indicates that it is very probably an extinct comet. Hence there is little doubt that some comets can evolve into asteroidal objects and that a few can be placed in orbits like those of the Earth-crossing asteroids. As found by Wetherill (1979) from Monte Carlo studies, when the orbit of P/Encke is chosen as a model starting orbit, further orbital evolution resulting from close encounters with the terrestrial planets produces an equilibrium distribution of orbits like that observed among the Earth-crossing asteroids.

The supply of comets which become extinct in orbits safe from Jupiter encounter appears to be adequate to maintain the population of Earth crossers. That one such comet is observed in the process of decaying into an Earth-crossing asteroid is evidently a matter of luck. On the average, only one comet like P/Encke is required every few tens of thousands of years to sustain the Earth-crossing asteroid population, whereas the average period of activity of short-period comets may be no greater than a few thousand years.

At least two Earth-crossing asteroids, 1978 SB and 1973 NA, are on "comet-like" orbits and may represent recent additions to the Earth-crossing asteroid population. The maximum aphelion of 1978 SB is 4.09 AU, like that of P/Encke, just inside the limit where it is safe from Jupiter encounter. As noted by Kresák (1977) the orbit of 1973 NA is comparable, in certain respects, to that of many periodic comets. There is a large periodic oscillation of eccentricity of 1973 NA; at times its aphelion exceeds 4.6 AU. Because of its high inclination and restricted range of ω, however, it is also safe from Jupiter encounter.

VI. EVIDENCE FOR FLUCTUATION OF THE EARTH-CROSSING ASTEROID POPULATION IN LATE GEOLOGIC TIME

The history of the Earth-crossing asteroid population is reflected in the geologic record of impact craters on the earth and on the moon. The population and associated impact rate must be translated into an equivalent cratering rate in order to interpret that record. To do this it is necessary to determine sizes and volumes for asteroids of a given magnitude and to make estimates of asteroid bulk densities. Kinetic energies of impact for asteroids of a given mass can then be solved, with the aid of Eq. (7), by taking proper account of the acceleration of the asteroids in the gravity fields of the earth and the moon. Finally, the kinetic energies are related to impact crater diameters through an appropriate scaling relationship.

Diameters of spherical bodies corresponding in brightness to asteroids of a given absolute magnitude are given by

$$\log d = 3.122 - 0.2V(1,0) - 0.5 \log p_V \quad (8)$$

where d is the diameter in km. The constant in this equation is based on $V = -26.77$ for the sun, as adopted by Gehrels *et al.* (1964). For asteroids of $V(1,0) = 18$, Eq. (8) reduces to

$$\log d_{18} = -0.5 \log p_V - 0.472. \quad (9)$$

We will evaluate the cratering rate for three different assumptions about the distributions of p_V in the Earth-crossing asteroid population:

1. All the Earth crossers are bright (mean $p_V = 0.14$, equal to the mean for *S*-type asteroids.
2. All the Earth crossers are dark (mean $p_V = 0.037$, equal to the mean for *C*-type asteroids).
3. Half of the Earth crossers are bright ($p_V = 0.14$) and half are dark ($p_V = 0.037$).

This set of assumptions more than spans the plausible range of mean albedos for the Earth crossers. At $p_V = 0.14$, $d = 0.89$ km and at $p_V = 0.037$, $d = 1.73$ km, for asteroids of absolute visual magnitude 18.

Bulk densities may be estimated on the basis of analogies drawn between various types of asteroids and meteorites. The material of *S*-type asteroids is here assigned a density of 3.5 g cm^{-3}, comparable to the density of ordinary chondrites, and that of *C*-type asteroids 2.5 g cm^{-3}, comparable to the density of carbonaceous chondrites. A correction to the density is then applied to account both for void space due to brecciation and for nonspherical shapes of the asteroids.

Many or most asteroids in the size range of Earth crossers may be expected to contain significant void space due to brecciation resulting from collisions. Generally about 25% new void space, the so-called bulking factor, is created by fragmentation of rock in mining and quarrying operations. Observed bulking factors for ejecta from cratering experiments in dense rock range from 4 to 70%, with most values in the range of 20 to 30% (Nugent and Banks 1966; Frandsen 1969). A volume expansion of 28% was adopted as a best value for the ejecta from solid rock by Ramspott (1970). Comparable or somewhat smaller amounts of excess void space occur in uncemented breccias beneath impact craters, as indicated by gravity observations on both Earth and Moon (Innes 1961; Dvorak 1979).

Observations of Phobos provide the only available check on void space, presumably due to brecciation, in a body close to the size of Earth-crossing asteroids. Photometric observations of Phobos show that it has a low albedo and a spectral reflectance like that of the *C*-type asteroids and certain types of carbonaceous meteorites (Pang *et al.* 1978; Pollack *et al.* 1978). Hence it is reasonable to suppose that Phobos is composed of material with a density of $\sim$2.5 g cm^{-3}, like that of the largest *C*-type asteroid, Ceres (Morrison 1976; see the chapter by Schubart and Matson), or the meteorites with comparable optical characteristics. From data presented by Mason (1963), the mean density of CI meteorites is found to be 2.33 g cm^{-3}, that of CII meteorites 2.73 g cm^{-3}, and that of CIII meteorites 3.48 g cm^{-3}. Only the CI and CII meteorites are similar optically to Phobos, Ceres, or other *C*-type asteroids. Therefore we adopted 2.5 g cm^{-3}, a density intermediate between that of CI and CII meteorites, as the best estimate for the density of the material of all *C*-type objects. But the estimated density of Phobos is 1.9 ± 0.6 g cm^{-3} (Tolson *et al.* 1978). While this estimate just overlaps our adopted density of 2.5 g cm^{-3}, at one standard deviation, it is interesting that the central value is 24% below the adopted density, about what would be expected for a small brecciated body. The observed distribution of craters on Phobos (Thomas *et al.* 1979; see the chapter by Veverka and Thomas) suggests to us that it is thoroughly fragmented. A void space of 24% of the total volume is here taken as the best estimate of the bulking factor due to impact brecciation of small asteroids.

An average correction for the nonspherical shapes of small asteroids, taking into account the types of irregularities observed on Phobos and the evidence from lightcurves for marked elongation of some Earth-crossing objects, is roughly estimated at 8%. This correction may be thought of as "external void space." The total correction is equivalent to a reduction of the bulk density by 32%. Final calculated masses are 0.87×10^{15} g for *S*-type asteroids and 4.5×10^{15} g for *C*-type asteroids at V(1,0) = 18.

TABLE IV

Estimates of Present Cratering Rate on Earth

Calculated production of craters ≥ 10 km diameter $10^{-14}km^{-2}yr^{-1}$		Last 1/2 Gyr record on North America $10^{-14}km^{-2}yr^{-1}$	
All objects bright (p_v = 0.14)	~ 1.5	$1.4_4 \pm 0.4$	(Grieve and Dence 1979)
Half bright, half dark	~ 2.3	2.2 ± 1.1	(Shoemaker 1977)
All objects dark (p_v = 0.037)	~ 3.5		
Equivalent cratering rate on Earth from last 3.3 Gyr record on Moon		0.3	(derived from Neukum *et al.* 1975)
		1.1 ± 0.5	(Shoemaker 1977)

Crater diameters are obtained from the scaling relationship

$$D_e = 74\, W^{\frac{1}{3.4}} \tag{10}$$

where D_e is the rim diameter of the crater on Earth in meters, W is the kinetic energy of the asteroid in kilotons TNT equivalent (1 kt TNT equivalent = 4.185×10^{19} ergs), which is based on nuclear cratering experiments (Shoemaker *et al.* 1963; Shoemaker 1977). For craters on Earth larger than about 3 km, D_e given by Eq. (10) is multiplied by 1.3 to account for crater collapse. This is a conservative correction for crater collapse; a best value may be closer to 1.35 (Shoemaker 1962; 1977). Use of Eq. (10), including the correction for collapse, yields crater diameters within 5% of those obtained from the scaling relationship of Dence *et al.* (1977) for terrestrial craters larger than 3 km. For craters on the moon, the diameters given by Eq. (10) are scaled for the difference in gravitational acceleration (Gault and Wedekind 1977) by

$$D_m/D_e = (g_e/g_m)^{\frac{1}{6}} \tag{11}$$

where D_m is the rim diameter of a crater on Moon in meters, g_e is the surface gravity on Earth and g_m is the surface gravity on the moon. Correction for crater collapse is not required for most lunar craters smaller than 15 km diameter.

Cratering rates on Earth for different assumptions about the distribution of p_v are given in Table IV. The calculated cratering rates are based on the collision rates of various classes of Earth-crossing asteroids listed in Table II and an rms impact velocity, weighted by collision probability, of 20.1 km sec^{-1} for all classes of Earth crossers. The cumulative frequency of craters produced by asteroid impact was assumed to be proportional to $D^{-1.7}$, consistent with the observed distribution of post-mare lunar craters larger than 3 km diameter (Shoemaker *et al.* 1963; Baldwin 1971).

Assuming that half of the Earth-crossing asteroids are similar in albedo to *S*-type asteroids and half are similar in albedo to *C*-type, the estimated present production of craters ⩾ 10 km diameter on Earth is ~ 2.3 × $10^{-14} km^{-2} yr^{-1}$. This cratering rate calculated from observations of Earth-crossing asteroids is essentially indistinguishable from the average production of impact craters in the last half billion years on North America estimated by Shoemaker (1977) from impact structures in the United States (Table IV). A somewhat lower rate, equivalent to $1.4_4 \pm 0.4 \times 10^{-14} km^{-2} yr^{-1}$ for craters ⩾ 10 km diameter, was found by Grieve and Dence (1979) for the Phanerozoic crater record of the structurally stable part of North America.

The cratering rates estimated from asteroid data and from the Phanerozoic geologic record in North America have comparable uncertainties. Both types of estimates are likely to be minimum values, as they depend on completeness of survey of regions sampled. The cratering rate estimate assuming half of the Earth-crossing asteroids are bright and half are dark and the estimate from the 0.5 Gyr North American cratering record by Shoemaker are twice as high as the equivalent average cratering rate on Earth derived from the density of craters on 3.3 Gyr old surfaces on the moon adopted by Shoemaker (1977). Corrections are made in deriving the terrestrial cratering rate from the lunar record for differences in the capture cross-sections of the earth and moon, for differences in crater scaling in the gravity fields of the two bodies, and for collapse of craters >3 km diameter on the earth. The estimate of long-term average cratering rate derived from the lunar record is a maximum, owing to the fact that craters older than 3.3 Gyr may be erroneously counted as formed on 3.3 Gyr lava surfaces (Neukum *et al.* 1975). Neukum *et al.* (1975) estimate a density for craters ⩾ 10 km diameter more than three times lower than the density adopted by Shoemaker (1977) for 3.3 Gyr lava surfaces. Hence the cratering rates estimated from the Earth-crossing asteroid population and the North American cratering record may be more than twice as high as the equivalent terrestrial cratering rate implied by the 3.3 Gyr lunar cratering record.

Because the estimates of the modern cratering rate obtained for the Earth from asteroid observations (which are consistent with the North America geologic record for the Phanerozoic) are conservative and are, therefore, minimum estimates, whereas the estimate by Shoemaker (1977) of the 3.3 Gyr average cratering rate obtained from the moon is a maximum and probably errs on the high side, the difference between the two probably should be regarded as significant. An increase in cratering rate in the last half billion years implies a corresponding increase in the population of Earth-crossing asteroids. If Earth-crossing asteroids are derived primarily from extinct comets, an increase in Earth-crossing asteroids suggests that there has been an increase in the flux of long-period comets crossing the orbit of Jupiter. A sudden increase in the Earth-crossing asteroid population might arise from a catastrophic collision in the asteroid belt that injected large numbers of fragments into one of the secular resonances or commensurabilities. Such a perturbation in the population would be expected to decay in times on the order of a few tens of millions of years, however, whereas the consistency between the present Earth-crossing asteroid population and the North American cratering record suggests the average level of the population has been near the present level for times of the order of several 100 million years. Hence a change in the comet flux seems a more likely explanation for the apparent change in Earth-crossing asteroid population.

Fluctuations in the flux of comets in the inner solar system probably reflect changes in the number or mass of stars passing near the Sun that perturb the Oort cometary cloud. On the basis of the difference between the late terrestrial cratering record and the 3.3 Gyr lunar record, we tentatively postulate that the average flux of stars in the solar neighborhood has been higher during the last several hundred million years than during the preceding ~ 3 billion years.

Acknowledgments. We thank B. G. Marsden for providing us with unpublished results from his numerical integration of the motion of 1915 Quetzalcoatl and for many other courtesies that he has extended to us in the course of this work. E. Roemer kindly provided unpublished photographic magnitudes of comets observed by her over a period of nearly ten years while she was at the Flagstaff station of the U.S. Naval Observatory. G. W. Wetherill and S. J. Weidenschilling made many helpful suggestions in reviewing this manuscript. Part of this research has been carried out at California Institute of Technology and part is the result of one phase of research carried out at the Jet Propulsion Laboratory, both under NASA contracts.

Note added in proof. The errors in calculating the rate of advance of the apsides and the node rate of earth-crossing asteroids are sufficiently large that the identification of asteroids occurring in the secular resonances should be treated with caution. At the present time it is not known with certainty whether any of the earth-crossing asteroids occur in secular resonances.

REFERENCES

Arnold, J. R. 1964. *The Origin of Meteorites as Small Bodies, Isotopic and Cosmic Chemistry*, N. Holland Publishing Co., (Amsterdam: North Holland Publ. Co.), pp. 347-364.

Arnold, J. R. 1965. The origin of meteorites as small bodies. 2. The model. *Astrophys. J.* 141: 1536-1547.
Baldwin, R. B. 1971. On the history of lunar impact cratering: The absolute time scale and the origin of planetesimals. *Icarus* 14: 36-52.
Bowell, E. 1979. 1978 SB. *IAU Circ.* 3284.
Bowell, E.; Chapman, C. R.; Gradie, J. C.; Morrison, D.; and Zellner, B. 1978. Taxonomy of Asteroids. *Icarus* 35: 313-335.
Bowell, E.; Chernykh, N. S.; Helin, E. F.; Kowal, C. T.; Marsden, B. G.; Niehoff, J. C.; Sebok, W. L.; Shoemaker, E. M.; Wetherill, G. W.; Williams, J. G.; and Zellner, B. 1979. Discovery and observations of asteroid 1976 UA. *Icarus* (to be submitted).
Danielsson, L., and Ip, W. H. 1972. Capture resonance of asteroid 1685 Toro by the earth. *Science* 176: 906-907.
Dence, M. R.; Grieve, R. A. F.; and Robertson, R. B. 1977. Terrestrial impact structures: Principal characteristics and energy considerations. In *Impact and Explosion Cratering: Planetary and Terrestrial Implications*, eds. D. J. Roddy, R. O. Pepin and R. B. Merrill (New York: Pergamon Press), pp. 247-275.
Dvorak, J. J. 1979. Analysis of small scale lunar gravity anomalies: Implications for crater formation and crustal history: Ph. D. dissertation, California Institute of Technology.
Frandsen, A. D. 1969. Engineering properties investigations of the Cabriolet Crater. *U.S. Army Engineers Nuclear Cratering Group Report* PNE-957.
Gault, P. E., and Wedekind, J. A. 1977. Experimental hypervelocity impact into quartz sand II. Effects of gravitational acceleration. In *Impact and Explosion Cratering*, eds. D. J. Roddy, R. O. Pepin, and R. B. Merrill (New York: Pergamon Press), pp. 1231-1244.
Gehrels, T.; Coffeen, T.; and Owings, D. 1964. Wavelength dependence of polarization. III. The lunar surface. *Astron. J.* 69: 826-852.
Grieve, R. A. F., and Dence, M. R. 1979. The terrestrial cratering record. II. The crater production rate. *Icarus* 38: 230-242.
Helin, E. F., and Shoemaker, E. M. 1977. Discovery of asteroid 1976 AA. *Icarus* 31: 415-419.
Helin, E. F., and Shoemaker, E. M. 1979. Palomar Planet-crossing Asteroid Survey, 1973-1978. *Icarus*, in press.
Helin, E. F.; Shoemaker, E. M.; and Wolfe, R. F. 1978. Ra-Shalom: Third member of the Aten class of Earth-crossing asteroids (abstract). *Bull Amer. Astron. Soc.* 10: 732.
Innes, M. J. S. 1961. The use of gravity methods to study the underground structure and impact energy of meteorite craters. *J. Geophys. Res.* 66: 2225-2239.
Ip, W. H., and Mehra, R. 1973. Resonances and librations of some Apollo and Amor asteroids with the earth. *Astron. J.* 78: 142-147.
Janiezek, P. M.; Seidelmann, P. D.; and Duncombe, R. L. 1972. Resonances and encounters in the inner solar system. *Astron. J.* 77: 764-773.
Jones, T. J., and Morrison, D. 1974. A recalibration of the radiometric/photometric method of determining asteroid sizes. *Astron. J.* 79: 892-895.
Kresák, L., 1977. Asteroid versus comet discrimination from orbital data. In *Comets, Asteroids, Meteorites*, ed. A. H. Delsemme (Toledo, Ohio: University of Toledo Press), pp. 313-321.
Lebofsky, L. A.; Lebofsky, M. J.; and Rieke, G. H. 1979. Radiometry and surface properties of Apollo, Amor, and Aten asteroids. *Astron. J.* 84: 885-888.
Lebofsky, L. A.; Veeder, G. J., Lebofsky, M. J.; and Matson, D. L. 1978. Visual and radiometric photometry of 1580 Betulia. *Icarus* 35: 336-343.
Marsden, B. G. 1970. On the relationship between comets and minor planets. *Astron. J.* 75: 206-217.
Mason, B. 1963. The carbonaceous chondrites. *Space Sci. Rev.* 1: 621-646.
Morrison, D. 1976. The densities and bulk composition of Ceres and Vesta. *Geophys. Res. Lett.* 3: 701-704.
Morrison, D. 1977. Asteroid sizes and albedos. *Icarus* 31: 185-220.
Morrison, D.; Gradie, J. C.; and Reike, G. H. 1976. Radiometric diameter and albedo of the remarkable asteroid 1976 AA. *Nature* 260: 691.

Neukum, G.; König, B.; Fechtig, H.; and Storzer, D. 1975. Cratering in the Earth-moon system: Consequences for age determination by crater counting. *Proc. Lunar Sci. Conf. VI* (Oxford: Pergamon Press), pp. 2597-2620.

Nugent, R. C., and Banks, D. C. 1966. Engineering-geologic investigations, Project Danny Boy. *U.S. Army Engineers Nuclear Cratering Group Report PNE-5005.*

Öpik, E. J. 1951. Collision probabilities with the planets and distribution of interplanetary matter. *Proc. Roy. Irish Acad.* 54A: 165-199.

Öpik, E. J. 1963. The stray bodies in the solar system. Part 1. Survival of cometary nuclei and the asteroids: *Advan. Astron. Astrophys.* 2: 219-262.

Öpik, E. J. 1976. *Interplanetary Encounters: Close-Range Gravitational Interactions.* (New York: Elsevier).

Pang, K. D.; Pollack, J. B.; Veverka, J.; Lane, A. L.; and Ajello, J. M. 1978. The composition of Phobos: Evidence for carbonaceous chondrite surface from spectral analysis. *Science* 199: 64-66.

Pollack, J. B.; Veverka, J.; Pang, K. D.; Colburn, D.; Lane, A. L.; and Ajello, J. M. 1978. Multicolor observations of Phobos with the Viking lander cameras: Evidence for a carbonaceous chondritic composition. *Science* 199: 66-69.

Ramspott, L. C. 1970. Empirical analysis of the probability of formation of a collapsed crater by an underground nuclear explosion. *Univ. of Calif. Radiation Lab. Rept. UCRL-50883* (classified).

Scholl, H., and Froeschlé, C. 1977. The Kirkwood gaps as an asteroidal source of meteorites. In *Comets, Asteroids, Meteorites,* ed. A. H. Delsemme (Toledo, Ohio: University of Toledo Press), pp. 293-295.

Sekanina, Z. 1971. A core-mantle model for cometary nuclei and asteroids of possible cometary origin. In *Physical Studies of Minor Planets,* ed. T. Gehrels (NASA SP-267, Washington, D.C.: U.S. Government Printing Office), pp. 423-426.

Sekanina, Z. 1972. A model for the nucleus of Encke's Comet. In *The Motion, Evolution of Orbits, and Origin of Comets,* eds. G. S. Chebotarev and Kazimirchak-Polonskaya (Dordrecht: D. Reidel), pp. 301-307.

Shoemaker, E. M. 1962. Interpretation of Lunar Craters. In *Physics and Astronomy of the Moon,* ed. Z. Kopal (New York: Academic Press), pp. 283-359.

Shoemaker, E. M. 1977. Astronomically observable crater-forming projectiles. In *Impact and Explosion Cratering: Planetary and Terrestrial Implications,* eds. D. J. Roddy, R. O. Pepin, and R. B. Merrill (New York: Pergamon Press), pp. 617-628.

Shoemaker, E. M.; Hackman, R. J.; and Eggleton, R. E. 1963. Interplanetary correlation of geologic time. *Adv. Astronaut. Sci.* 8: 70-89.

Shoemaker, E. M., and Helin, E. F. 1977. Populations of planet-crossing asteroids and the relation of Apollo objects to main-belt asteroids and comets. In *Comets, Asteroids, Meteorites,* ed. A. H. Delsemme (Toledo, Ohio: University of Toledo Press), pp. 297-300.

Shoemaker, E. M., and Helin, E. F. 1978. Earth-approaching asteroids: Populations, origin and compositional types. NASA Conf. Public. 2053, pp. 161-175.

Thomas, P.; Veverka, J.; and Chapman, C. R. 1979. Crater densities on the satellites of Mars. Submitted to *J. Geophys. Res.*

Tolson, R. H.; Duxbury, T. C.; Born, G. H.; Christensen, E. J.; Diehl, R. E.; Farless, D.; Hildebrand, C. E.; Mitchell, R. T.; Molko, P. M.; Morabito, L. A.; Palluconi, F. O.; Reichert, R. J.; Taraji, H.; Veverka, J.; Neugebauer, G.; and Findlay, J. T. 1978. Viking first encounter of Phobos: Preliminary results. *Science* 199: 61-64.

Wetherill, G. W. 1967. Collisions in the asteroid belt. *J. Geophys. Res.* 72: 2429-2444.

Wetherill, G. W. 1976. Where do the meteorites come from: A reevaluation of the Earth-crossing Apollo objects as sources of stone meteorites. *Geochim. Cosmochim. Acta.* 40: 1297-1317.

Wetherill, G. W. 1977. Fragmentation of asteroids and delivery of fragments to Earth. In *Comets, Asteroids, Meteorites,* ed. A. H. Delsemme (Toledo, Ohio: University of Toledo Press), pp. 283-291.

Wetherill, G. W. 1979. Steady state populations of Apollo-Amor objects. *Icarus* 37: 96-112.

Wetherill, G. W., and Williams, J. G. 1968. Evaluation of the Apollo asteroids as sources

of stone meteorites. *J. Geophys. Res.* 73: 635-648.
Wetherill, G. W., and Williams, J. G. 1979. Origin of differentiated meteorites. In *Proc. 2nd Internat. Conf. on Origin and Abundance of the Elements*, ed. H. de la Roche (New York: Pergamon Press). In press.
Williams, J. G. 1969. Secular perturbations in the solar system: Ph.D. dissertation, University of California at Los Angeles.
Williams, J. G. 1971. Proper elements, families, and belt boundaries. In *Physical Studies of Minor Planets*, ed., T. Gehrels (NASA SP-267, Washington, D.C.: U.S. Government Printing Office), pp. 177-181.
Williams, J. G. 1973*a*. Meteorites from the asteroid belt? (abstract) *Eos: Trans. Amer. Geophys. Union* 54: 233.
Williams, J. G. 1973*b*. Secular resonances (abstract). *Bull. Amer. Astron. Soc.* 5: 363.
Williams, J. G. 1979. Classification of planet-crossing asteroids (abstract). *Lunar Sci. X.* Lunar and Planet. Inst. p. 1349.
Williams, J. G., and Wetherill, G. W. 1973. Physical studies of the minor planets. XIII. Long-term orbital evolution of 1685 Toro. *Astron. J.* 78: 510-515.
Zellner, B.; Gehrels, T.; and Gradie, J. 1974. Minor planets and related objects. XVI. Polarimetric parameters. *Astron. J.* 79: 1100-1110.
Zimmerman, P. D., and Wetherill, G. W. 1973. Asteroidal source of meteorites. *Science* 182: 51-53.